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Frequency Analysis of Skew Plate having Elliptical Thickness

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ABSTRACT: In this work, vibrational behaviour of frequency of nonuniform skew (parallelogram) plate under temperature field is carried out. For nonuniformity consideration, author introduced elliptical variation in thickness, which is new variation introduced by authors. For temperature field, authors considered bi-linear temperature variat.....

Key Words: Skew Plate; Frequency; Elliptical Thickness; Temperature.

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1. Introduction

Research on plate vibration has demonstrated that geometric nonuniformity and thermal effects play a crucial role in governing the dynamic response of structural components. Previous studies have primarily focused on linear, parabolic, exponential, and circular thickness variations to control the frequency behavior of parallelogram plates. However, these conventional profiles provide limited effectiveness in reducing natural frequencies. To address this gap, the present study introduced elliptical variation to enhance vibration control relative to earlier approaches. es Domagalski and Uhl [7] explored how changing plate thickness can be used to maximize frequency gaps. Through finite-element simulations combined with optimization algorithms, they demonstrated that a properly designed taper can widen the se

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Sharma and Sharma [10] analyzed free vibrations of orthotropic rectangular plates using the Rayleigh–Ritz energy method with admissible polynomial functions. Their study incorporated two-dimensional thickness and temperature variations, demonstrating that an increase in temperature gradient and taper ratio led to reduced natural frequencies, thereby offering benchmark data for thermally sensitive plate designs. Rahbar-Ranji and Shahbaztabar [11] employed the Rayleigh–Ritz approach to investigate orthotropic plates on Pasternak-type elastic foundations. They showed that foundation stiffness and shear-layer parameters significantly affect modal characteristics, with higher stiffness resulting in increased fundamental

In the present study, authors focussed to discuss the effect of elliptical thickness variation (new developed variation) variable temperature on frequencies of parallelogram plate under bilinear temperature effect given in Table 1.

2. Formulation and analysis

This section presents the governing differential equation of the system, mathematical formulation and their solutions.

2.1. Motion for isotropic parallelogram plate with variable flexural rigidity

The differential equation of motion for the plate and the expression for bending moment intensity M_ζ, M_ψ and twisting moment intensity $M_{\zeta\psi}$ in skew coordinates system are given as

$$\frac{\partial^2 M_\zeta}{\partial \zeta^2} - 2\,\tan\theta \frac{\partial^2 M_{\zeta\psi}}{\partial \zeta^2} + 2\,\sec\theta \frac{\partial^2 M_{\zeta\psi}}{\partial \zeta \partial \psi} + \tan^2\theta \frac{\partial^2 M_\psi}{\partial \zeta^2} - 2\,\sec\theta\tan\theta \frac{\partial^2 M_\psi}{\partial \zeta \partial \psi} + \sec^2\theta \frac{\partial^2 M_\psi}{\partial \psi^2} = \rho\, l\, \frac{\partial^2 \phi}{\partial t^2}. \tag{2.1}$$

Eq. (2.1) represents the equation of motion of isotropic parallelogram plate with variable D_1 (flexural rigidity).

3. Formulation of the Problem

Consider a skew (parallelogram) plate made with non-homogeneous material property with variable thickness l having skew angle θ, length a, breadth b, density ρ and Poisson's ratio ν as shown in Figure 1.

4. Computation of Natural Frequencies

In order to find the frequency equation, authors used Rayleigh Ritz technique which is taken as in [16]:

5. Numerical results and discussions

Table 1 reports the frequency modes of the plate for increasing values of the tapering parameter, which varies from 0.1 to 0.9 in steps of 0.2, under different thermal gradient conditions(i.e., $\alpha = 0.1, 0.3, 0.5, 0.7$). It is observed that the frequency modes consistently increase with an increase in the tapering parameter. The corresponding percentage increases in frequency modes for the considered thermal gradients are 9.25 %, 9.41%, 9.57%, and 9.75%, respectively. These results indicate that higher temperature gradients lead to a greater rate of increase in the frequency modes, demonstrating the combined influence of tapering and thermal effects on the vibrational behaviour of the plate.

6. Comparisons of frequency modes

In this section, authors discussed about the comparison of frequencies obtained in present study with the frequency reported by [18], [19], [20]. The results are displayed in graphical form. Here, authors also provided the percentage difference in obtained frequency modes with the frequency calculated by [18], [19],

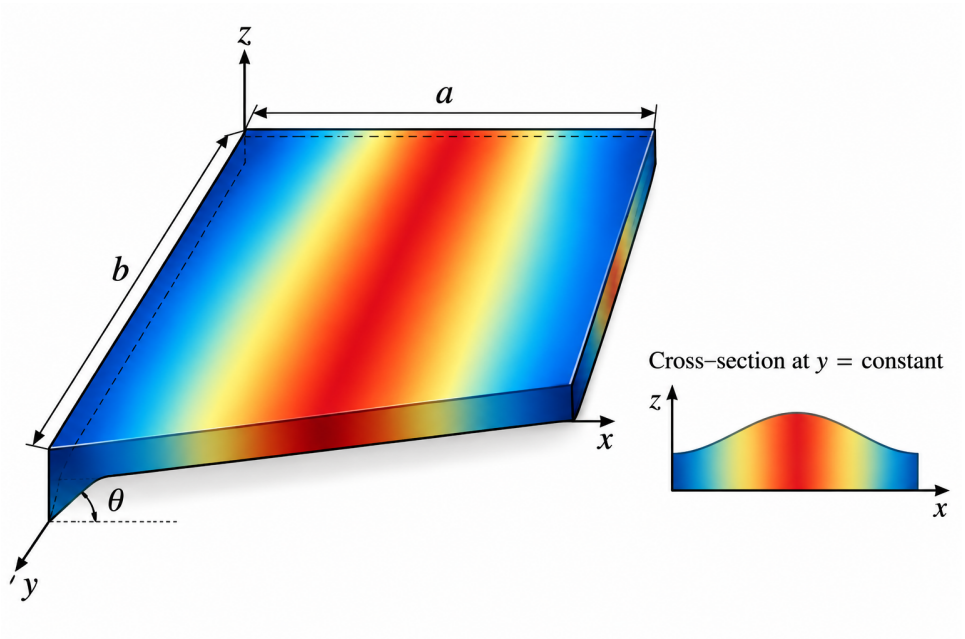


Figure 1: Skew plate having one-dimensional elliptical thickness variation.

Table 1: Frequency of skew plate corresponding to β

	$\alpha = 0.1$		$\alpha = 0.3$		$\alpha = 0.5$		$\alpha = 0.7$	
β	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0.1	73.89	293.86	72.02	286.46	70.11	278.87	68.14	271.06
0.3	75.53	299.72	73.65	292.26	71.72	284.60	69.74	276.73
0.5	77.22	305.75	75.32	298.22	73.38	290.49	71.38	282.55
0.7	78.95	311.94	77.04	304.34	75.08	296.54	73.06	288.53
0.9	80.73	318.28	78.80	310.60	76.82	302.73	74.79	294.66

7. Conclusions

This study aims to achieve effective control and optimization of the vibrational frequency modes. Based on the data and discussion, the authors concluded that the frequency modes are more optimized

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